

-4 -

the total # states with excess S plus the total # of states with excess $-S$, i.e.

$$it's = \underbrace{g(N, -S) + g(N, S)}_{\frac{2N!}{(\frac{1}{2}N+S)! (\frac{1}{2}N-S)!}} = g(N, S) + g(N, -S) =$$

(b) Now apparently we imagine that we constrain the length of the poly to ℓ and would like to compute the associated entropy. Mathematica this is not that challenging, but there ~~are~~ is a good reason to believe that the task, especially in connection with trying to find the form makes no sense. Please read the attached explanation. In any case, we take the log of the multiplicity above, where $\ell = 2g(N, S)$.

$$S = \log(2N!) - \log((\frac{1}{2}N+S)!) - \log((\frac{1}{2}N-S)!) =$$

Stirling \approx

$$\begin{aligned} &= \log(2N!) - (\frac{1}{2}N+S) \log(\frac{1}{2}N+S) + (\frac{1}{2}N+S) - (\frac{1}{2}N-S) \log(\frac{1}{2}N-S) + (\frac{1}{2}N-S) \\ &= \log(2N!) - (\frac{1}{2}N+S) \log\left(\frac{1}{2}N\left(1+\frac{2S}{N}\right)\right) - (\frac{1}{2}N-S) \log\left(\frac{1}{2}N\left(1-\frac{2S}{N}\right)\right) + \cancel{\frac{N}{2}} \\ &= \log(2N!) - \left(\frac{1}{2}N+S\right) \left[\log\left(\frac{1}{2}N\right) + \log\left(1+\frac{2S}{N}\right) \right] - \left(\frac{1}{2}N-S\right) \left[\log\left(\frac{1}{2}N\right) + \log\left(1-\frac{2S}{N}\right) \right] \\ &\text{log}(1+x) \approx x \\ &= \log(2N!) - \left(\frac{1}{2}N+S\right) \log\left(\frac{1}{2}N\right) - \left(\frac{1}{2}N+S\right) \frac{2S}{N} - \left(\frac{1}{2}N-S\right) \log\left(\frac{1}{2}N\right) - \left(\frac{1}{2}N-S\right) \left(-\frac{2S}{N}\right) \\ &= \log(2N!) - 2\left(\frac{1}{2}N \log\left(\frac{1}{2}N\right) - \frac{1}{2}N\right) - \frac{4S^2}{N} = \\ &= \log(2N!) - 2\left(\log\left(\frac{1}{2}N!\right)\right) - \frac{4S^2}{N} = \end{aligned}$$

I'm not sure what happens to a factor of 2.

$$= \log\left(\frac{2N!}{\frac{1}{2}N! \frac{1}{2}N!}\right) - \frac{S^2}{N} = \underbrace{\log(2g(N, 0)) - \frac{\ell^2}{Np^2}}$$